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THE HOUSING MARKET IN THE DORTMUND REGION: A MICRO SIMULATION

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1. INTRODUCTION

The work on the simulation model of the Dortmund housing market is part of a larger research project conducted at the Institute of Urban and Regional Planning of the University of Dortmund within the Sonderforschungsbereich 26 Raumordnung und Raumwirtschaft, Münster, of the Deutsche Forschungsgemeinschaft. This ongoing project is aimed at the investigation of the relationships between economic, i.e. sectoral and technological, change, locational choice, mobility, and land use in urban regions. For this purpose, a spatially disaggregate dynamic simulation model of regional development was designed to simulate

- location decisions of industry, residential developers, and households,
- the resulting migration and commuting patterns,
- the land use development, and
- the impacts of public programs and policies in the fields of regional development, housing, and infrastructure.

It was decided to use the urban region of Dortmund as a study region, including Dortmund (pop. 630.000) and 19 neighbouring communities with a total population of 2.4 million (cf. Schönebeck, Wegener, 1978).

The intraregional migration component of this model is the housing market model described in this paper. The decision to model intraregional migrations as transactions on the regional housing market was based on the evidence established by many surveys that household mobility within urban regions, unlike long-distance mobility, is almost exclusively determined by housing considerations, i.e. by the changing housing needs of households during their life cycle. Accordingly, the housing market model developed is a microanalytic model of choice behaviour of households and landlords subject to economic and noneconomic choice restrictions.

2. MODEL HYPOTHESES

The *housing market* is the place where households trying to satisfy their housing needs interact with landlords trying to make a profit from earlier housing investments. Following this narrow definition, housing investment decisions are not part of the housing market, but are effected on the *land and construction market*, which is separate, but closely related to it. On the land and construction market housing has to compete with other kinds of land use.

Households and landlords are thus the principal actors of the housing market model. The design of the model was based on the following hypotheses about their behaviour:

- The housing demand of a household depends mainly on its position in its life cycle and its income.
- The satisfaction of a household with its housing situation can be represented by a utility function with the dimensions *housing size and quality*, *neighbourhood quality*, *location*, and *housing cost*.

- The willingness of a household to move is related to its dissatisfaction with its housing situation. A household willing to move actually does move if it finds a dwelling that gives it significantly more satisfaction than its present one.
- After a number of unsuccessful attempts to find a dwelling a household reduces its demand or abandons the idea of a move.
- Households have only limited information of the housing market; this limitation is related to their education and income.
- There are on the housing market local as well as social submarkets which are separated by economic and noneconomic barriers.
- Supply on the housing market is highly inelastic: There is practically no price adjustment in short market periods; quantity adjustment is delayed by long construction times.

In general, the housing market, although strongly regulated, fails to satisfy the housing needs of all groups of the population; instead, it tends to reinforce the spatial segregation of social groups.

3. MODEL STRUCTURE

Changes to the household and housing stock of an urban region can be caused by *time (aging)*, *migration*, *public programs*, or *private construction*. In the simulation model these four kinds of changes are executed in four separate submodels. The last two of them deal with changes of land use and the building stock, i.e. with the land and construction market. They will not be treated in this paper. The first two contain the housing market model. They will be described in the following two sections.

a. *The Aging Submodel*

In the housing market model changes of households and housing of the model region, *excluding* public programs and private construction, are simulated over a number of discrete time intervals or *periods*.

The model region consists of the labour market region of Dortmund including Dortmund itself with its ten urban districts plus ten neighbouring communities, as well as of nine residential communities outside of the labour market region. Thus the model region is subdivided into 29 *zones*.

The population of each zone is represented in the model as a distribution of households classified by

- nationality (native, foreign),
- age of head (16-29, 30-59, 60+ years),
- income (none, low, medium, high),
- size (1, 2, 3, 4, 5+ persons).

Similarly, housing of each zone is represented as a distribution of dwellings classified by

- type of building (single-family, multi-family),
- tenure (owner-occupied, rented, public),
- quality (very low, low, medium, high),
- size (1, 2, 3, 4, 5+ rooms).

All changes of population and housing during the simulation are computed for these 120 household types and 120 housing types. However, these household and housing types are collapsed to about 30 household and housing types for use in the occupancy matrix.

The *occupancy matrix* R of a zone represents the association of households with housing in the zone. Each element of the matrix contains the number of households of a certain type occupying a dwelling of a certain type, the total matrix contains all households occupying a dwelling or all dwellings occupied by a household (Fig. 1).

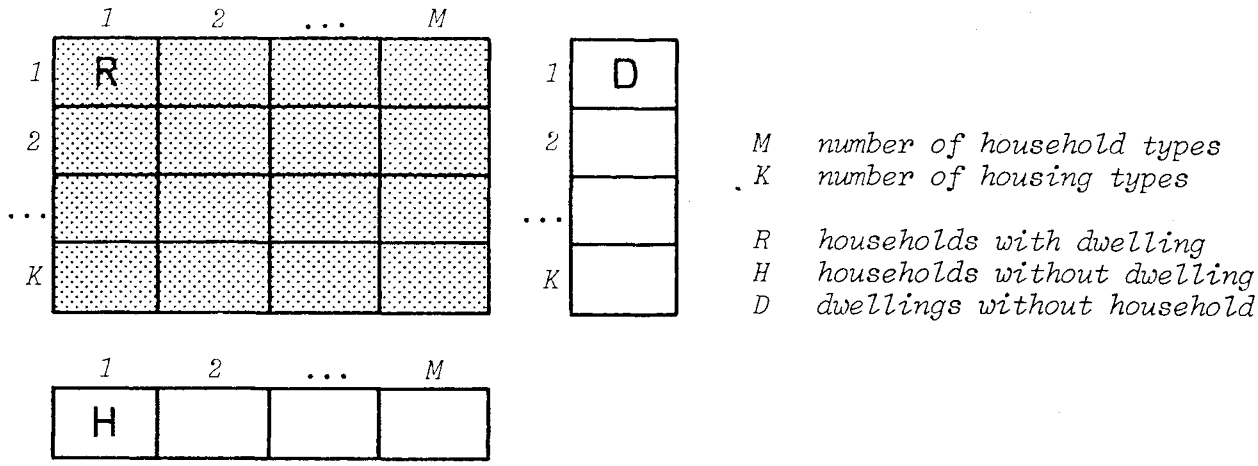

Fig. 1. Households and housing of a zone.

In addition, there exist for each zone a vector H of households currently without a dwelling and a vector D of dwellings currently without a household, i.e. vacant. H should contain zeros at the outset of each simulation period, but in D there may be vacant dwellings left over from previous periods.

All changes occurring to households and housing of a zone during a simulation period can be represented by movements into or out of or within the R matrix and the H and D vectors.

In the first, the *aging* submodel all changes of households and dwellings are computed which are assumed to result from biological, technological, or long-term socioeconomic trends originating outside of the model, i.e. which in the model are merely time-dependent. For households this includes demographic changes of household status in the life cycle such as birth, aging, death, marriage, and divorce, and all new or dissolved households resulting from these changes, as well as change of nationality or income. On the housing side it includes deterioration and certain types of rehabilitation and demolition. However, all changes of housing occupancy connected with migration decisions are left to the subsequent *migration* submodel.

In reality, both kinds of changes are mostly based on individual decisions and occur in a continuous stream of closely interrelated events. However, it is much more convenient to model them separately, each with a different type of model. Of course, that means that feedback between both kinds of changes is ignored, but that seems to be allright as housing decisions are assumed to depend on household status and income, and not conversely. The aging submodel therefore updates or "ages" households and dwellings by one simulation period *without* moving them relative to each other. This is accomplished by a Markov model with dynamic transition rates.

A transition rate is defined as the probability that a household or dwelling of a certain type changes to another type during the simulation period. The transition rates are computed as follows: The time-dependent changes to be simulated are interpreted as *events* occurring to a household or dwelling with a certain probability in a unit of time. These *basic event probabilities* and their expected future development are exogenously determined. Fifteen basic event probabilities have been identified for each of the three household age groups:

1 change of nationality,
2 aging,
3 marriage,
4 birth, native,
5 birth, foreign,
6 relative joins household,
7 death,
8 death of child,

9 marriage of child,
10 new household of child,
11 divorce,
12 rise of income,
13 decrease of income,
14 retirement,
15 new job,

and three for the four housing quality groups:

1 deterioration,
2 rehabilitation,
3 demolition.

Not all household events occur to every household. Some are applicable only to singles, some only to families, some only to adults, some only to children. The demographic event probabilities are checked against regionwide population projections and corrected if necessary. Some household events are followed by housing events, and vice versa: where a household dissolves, a dwelling is vacated, and where a nonvacant dwelling is demolished, a household is left without dwelling. The housing events contain only those changes of the housing stock which can be expected to occur under normal conditions in any housing area, i.e. a normal rate of deterioration, maintenance, rehabilitation, and demolition. More rehabilitation and demolition may occur later in the *private construction* submodel: rehabilitation as a response of housing investors to the demand situation observed on the housing market, demolition where housing has to make way for industrial or commercial land uses. In addition, rehabilitation and demolition may occur in the course of public construction programs in the *public programs* submodel.

The basic event probabilities are then aggregated to transition rates P for households and Q for dwellings using the disaggregate (120-type) household and housing distributions of each zone. Most events are independent of each other and can be aggregated multiplicatively; but some exclude others, i.e. are the complement of each other. Multiplication of the occupancy matrix R with the transition rate matrices P and Q yields the occupancy matrix aged by one simulation period (Fig. 2). This implies the assumption that all households of a certain type share the same transition rates, no matter in which dwelling they live, and vice versa.

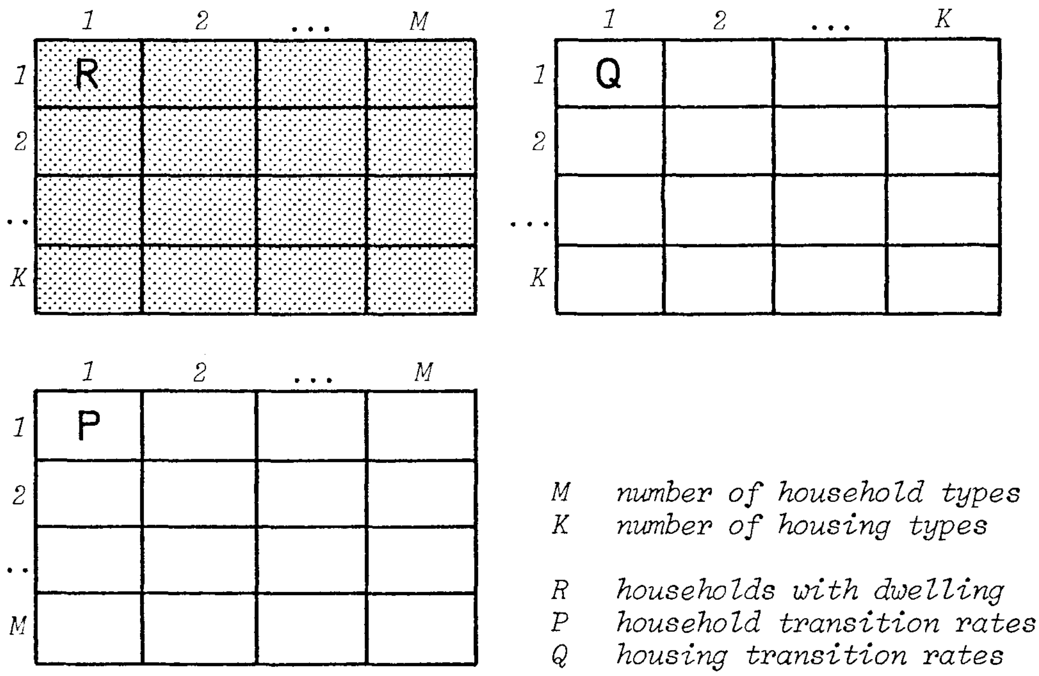

Fig. 2. Aging of households and housing of a zone.

Special provisions are necessary for events which modify the total number of households or dwellings of a zone. Such events are birth, marriage, marriage of child, divorce, death, new household of child, or demolition of a dwelling. Some of these events create a household without dwelling or a vacant dwelling, i.e. require a change in the H or D vectors. Moreover, also households without dwelling get older and vacant dwellings deteriorate or may be rehabilitated or be torn down, i.e. the H and D vectors themselves have to be aged.

b. *The Migration Submodel*

In the second, the *migration* submodel intraregional migration decisions of households are simulated. Migration is defined as a change of location of a household encompassing a change of residence. Consequently, the intraregional migration model is the actual housing market model.

When the migration submodel is entered, the following situation exists: All households and dwellings of all zones have been aged by one simulation period, i.e. now have the time label of the end of the current simulation period. However, no household has yet moved to another dwelling. That is to say: All households have proceeded in their life cycle - they have become older, children may have been born, the family income may have increased - , but their dwellings are still the same or even have deteriorated. Moreover, the expectations of the households with respect to size, quality, and location of housing generally will have increased. It may be assumed that many households which were quite content with their housing situation at the end of the last simulation period now are dissatisfied with it and are willing to improve it.

These households are the potential movers of the current market period. They are contained in the R matrix of each zone. Besides, there are households without dwellings contained in the H vector of each zone consisting of newly founded households looking for a dwelling and of households which unvoluntarily had to vacate their dwelling for various reasons. It is assumed that these households must get a dwelling during this market period.

In addition, there are two exogenously specified vectors of households: the vector H' containing households migrating into the region from elsewhere during the simulation period, and the vector H" containing households migrating out of the region. Both vectors have been aged already by another part of the model not discussed here in order to make them compatible with the households aged in the aging submodel. Inmigrant households are treated just like households without dwelling, except that they do not come from a particular zone. Outmigrant households are of interest because they vacate a dwelling.

On the housing side the situation is simpler. A dwelling can either be occupied or be vacant. In the first case it is contained in the R matrix, in the second case in the D vector of its zone. At the outset of the market period the D vector contains vacant dwellings left over from previous periods plus dwellings vacated by dissolved households during the current period. In addition, newly constructed dwellings which were begun in earlier periods may now have been completed and are entered into the D vector.

The R matrix and the H and D vectors of each zone, plus the H' and H" vectors are a complete representation of households and housing at the outset of the market simulation. Of these the H vectors of the zones and the H' vector clearly represent housing demand, and the D vectors of the zones and the H" vector clearly represent the supply side. The R matrices of the zones represent some of both because of the linkage between housing supply and housing demand by vacant dwellings being put on the market with each move. But which of the households in the R matrices will actually move during the market period is not known at this moment.

Fig. 3 illustrates this configuration. Unlike in the aging submodel, now the information of all zones has to be available simultaneously. Therefore, by the additional zonal dimension, the R matrix becomes three-dimensional, and the

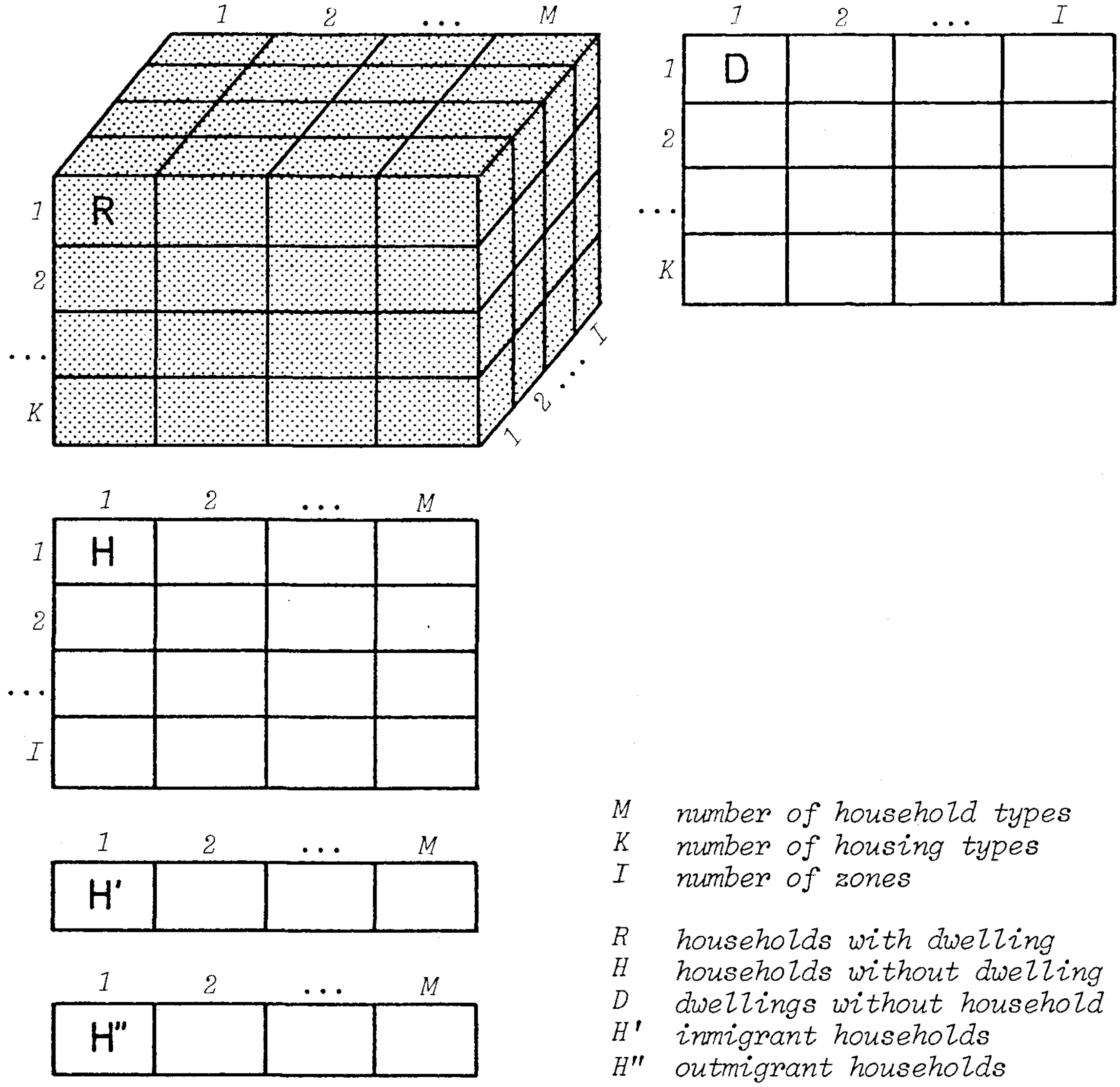

Fig. 3. Households and housing in the housing market model.

H and D vectors become two-dimensional matrices. In all vectors and matrices the collapsed or aggregate household and housing structure with about 30 types each is used.

The *satisfaction* of a household with its housing situation is represented in the model by a multidimensional preference function containing the dimensions *housing size and quality*, *neighbourhood quality*, *location*, and *housing cost*. Two of these four dimensions are themselves multiattribute:

- *Housing size and quality* is composed of the attributes defining a housing type: type of building, tenure, quality, size.
- *Neighbourhood quality* is composed of attributes selected or aggregated from state variables of the zones. There are some 300 state variables from the fields of population, employment, buildings, public facilities, transportation, and land use maintained and kept available on a file. In addition, accessibility measures indicating the location of the zone to the work places and to retail, education, and recreation facilities in other zones have been computed and are also kept on the file.

Evaluation and aggregation of the attributes is performed with the help of an additive multiattribute utility theory (MAUT) model. The general form of the utility function specified by this model is

$$A_{in} = \sum_m w_{mn} \, v_{mn}(a_{im}) \tag{1}$$

where A_{in} is the attractivity of evaluation object i for activity n, a_{im} is the attribute m of that evaluation object, and w_{mn} and v_{mn} are importance weights and value or utility functions, respectively, of attribute m as seen by actor type n. In the housing market model the actors are households, and the evaluation objects are dwellings or zones. Dwelling attributes are the attributes defining a housing type. Zonal attributes can be indicators for amenities supplied in the zones themselves or accessibility measures:

$$a_{im} = f_m(s_{ik}) \tag{2a}$$

$$a_{im} = \sum_j \frac{s_{jk}/f_m(c_{ij})}{\sum_j s_{jk}/f_m(c_{ij})} c_{ij} \tag{2b}$$

where $f_m(s_{ik})$ is a generation function specifying how to calculate a_{im} from variables s_{ik} of the zones, and $f_m(c_{ij})$ is a transformation of travel times or cost between zones i and j.

Obviously, there are as many preference systems as there are household types, as the household types have different housing needs depending on their size, position in their life cycle, and income. The preference systems of household types therefore differ in their attributes, utility functions, and weights. Moreover, the preference systems change in time, as aspiration levels rise and new priorities come into sight, and these changes are different for different household types. The model allows to specify different attributes, utility functions, and weights for different household types and different points in time.

The remaining two dimensions of housing satisfaction have only one attribute. The *location* dimension is represented by the attribute "job accessibility". A typical utility function of job accessibility looks like that in Fig. 4, left side. The explicit consideration of job accessibility in the model takes account of the fact that it is the most important location variable and perhaps the only one which really restricts the choice of a housing location. The second single-attribute dimension is *housing cost*. Its only attribute is rent or housing price plus housing operating cost in relation to (percent of) rent paying ability. Its utility function looks like that in Fig. 4, right side.

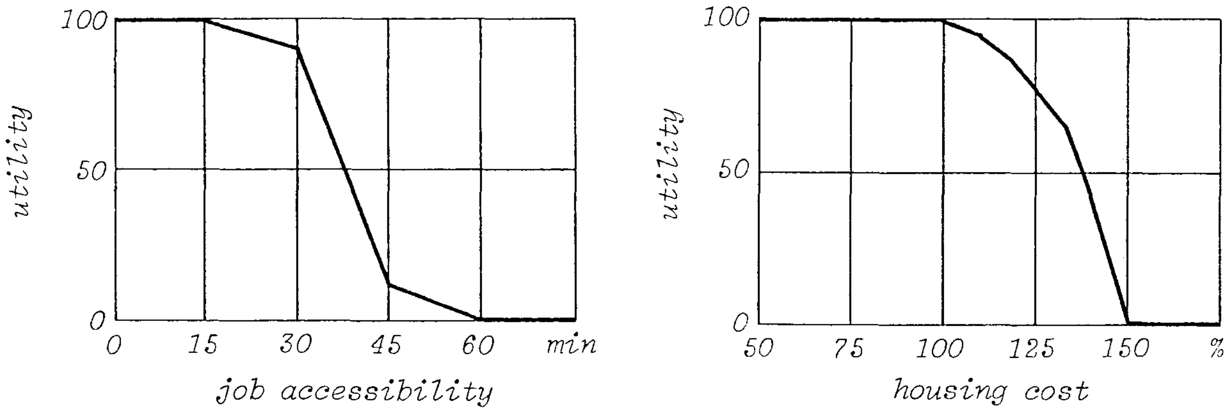

Fig. 4. Sample utility functions of job accessibility and housing cost.

For use in the housing market simulation the four dimensions of housing satisfaction are computed in advance and stored in two matrices: For each combination of household type, dwelling type, and zone, i.e. for each element of the three-dimensional occupancy matrix R, an *index of housing satisfaction* U_{mki} is calculated as a weighted aggregate of the four dimensions. Obviously, in this index only a general measure of job accessibility like that in (2b) can be included. Therefore an additional location measure is calculated for each pair of zones:

$$W_{ii'} = \sum_j \frac{T_{ij}\ T_{ji'}}{\sum_j T_{ij}\ T_{ji'}}\ v(c_{ji'}) \qquad (3)$$

where T_{ij} are home-to-job trips from i to j, $T_{ji'}$ are job-to-home trips from j to i', and $v(c_{ji'})$ is the utility function of job accessibility. That is, $W_{ii'}$ expresses the attractivity of zone i' as a new housing location with respect to job accessibility for a household now living in zone i whose head has a job in zone j. The measure $W_{ii'}$ is called the *migration distance* between i and i'.

With the matrices R, H, D, U, and W, and the two vectors H' and H" all necessary information is available to enter the housing market simulation, i.e. the simulation of the market clearing process.

This simulation presents several methodological problems. *First*, it is not known at this time which households of the R matrix will eventually decide to move as that certainly depends on the housing supply offered to them on the market. The housing supply, however, is only partly known because the major part of it consists of dwellings released by moving households during the market process, i.e. depends on just those decisions which are yet unknown. That is to say, the housing market, unlike many other markets, represents a complicated system of chain exchanges. *Second*, the level of information of the market actors about the housing market is generally low. That is, households inspect only a relatively small section of the market before making or not making a decision. *Third*, during a short market period the housing market is inelastic, and there is no real bidding process: A dwelling put on the market will not be given to the household bidding highest for it, but the landlord will select a household following a "first come, first served" rule or following other criteria, some of them noneconomic.

The simulation technique selected to deal with these problems is the Monte Carlo technique. It is based on the notion that the total market process can be sufficiently approximated by simulating a representative sample of individual market transactions. To achieve this, the model consists of a sequence of random selection operations by which hypothetical market transactions are generated. The random selection process is controlled by probability distributions which insure that only likely transactions are selected.

This stochastic kind of simulation has many advantages. Like no other technique it makes it possible to consider simultaneously objective and subjective, economic and noneconomic determinants of the individual decision situation of migrating and nonmigrating households, as well as their restricted information and choice on the housing market. In addition, the technique makes it easy to incorporate psychological hypotheses about the behaviour of households following successful or unsuccessful search experiences. In particular, it makes it possible to model the choice households make between intraregional migration and commuting. Probably the most important advantage of the technique, however, is that it solves the problem of modelling chain exchanges on the housing market in a simple and straightforward manner.

Micro simulation techniques have only recently been introduced into housing market modelling. Several models use probabilistic approaches to model the aging process of households and housing (e.g. Kain *et al*, 1976; Schacht, 1976). However, with the exception of one early example (Azcarate, 1970), no stochastic models of household decision behaviour have been reported.

The basic unit of the Monte Carlo simulation is the *market transaction*. A market transaction is any successfully completed operation by which a migration occurs, i.e. a household moves into or out of a dwelling or both. There are two ways to start a market transaction: A household decides to look for a dwelling

(*"dwelling wanted"*), or a landlord decides to offer a dwelling (*"dwelling for rent/sale"*). In either case the transaction may result in different kinds of migration: The household may be leaving the region (*"outmigration"*) or entering it (*"inmigration"*), or currently be without dwelling (*"new household/forced move"*), or occupying one (*"move"*). For the landlord offering a dwelling only the last three migration types are of interest.

The model starts by selecting a transaction type and a migration type. It is assumed that "dwelling wanted" and "dwelling for rent/sale" are equally likely to occur. The migration type is selected in proportion to the number of migrations to be completed of each type, i.e. the totals of H", H', and H for the first three migration types, respectively. For the fourth or "move" type a tentative estimate of the number of moves as a portion of the R matrix must be provided.

Once the transaction type and the migration type have been determined, the remaining parameters of the transaction are selected. A transaction has been completely defined if the following six parameters are known:

- m household type
- k old housing type
- i old zone
- j zone of job
- k' new housing type
- i' new zone

A move, for instance, is the migration of a household of type m which occupies a dwelling of type k in zone i and whose head works in zone j, into a dwelling of type k' in zone i'. Not all six parameters are required for all migration types: Obviously, no k can be specified for households without dwelling, nor can k and i for inmigrant households, but it is assumed that inmigrant households have a job in j already. Of outmigrant households only m, k, and i are of interest.

The sequence of selection steps performed for each combination of transaction type and migration type is shown in Fig. 5. In each step one additional parameter is determined until the transaction has been completely defined. The following example illustrates this: In the case of a household considering a move ("dwelling wanted"/"move"), first the household by type, zone, and dwelling type is selected with

$$p_{k|mi} = R_{mki}\ (100-U_{mki})^a \ / \sum_k R_{mki}\ (100-U_{mki})^a \qquad (4)$$

being the probability of dwelling type k to be selected if household type m and zone i are already known, which is to say that households which are dissatisfied with their housing situation are selected more often than others. In the next two steps it is asked in which zone j the head of the household might have his job and how this may restrict the choice of a new housing zone. With the help of the *migration distance* defined in (3) these two selection steps can be collapsed into one with

$$p_{i'|mki} = \sum_{k'} D_{k'i'}\ W_{ii'}^{\ b} \ / \sum_{i'} \sum_{k'} D_{k'i'}\ W_{ii'}^{\ b} \qquad (5)$$

being the probability of zone i' to be selected as a new housing zone where m, k, and i are given and zone j assumed to be the work place zone of the household head. In the final selection step the household attempts to find a dwelling in zone i' with

$$p_{k'|mkii'} = D_{k'i'}\ U_{mk'i'}^{\ a} \ / \sum_{k'} D_{k'i'}\ U_{mk'i'}^{\ a} \qquad (6)$$

being the probability of dwelling type k' to be selected if all other parameters are given.

start of transaction						
dwelling wanted				**dwelling for rent / sale**		
outmigration	inmigration	new household forced move	move	inmigration	new household forced move	move
household of type m	household of type m	household of type m	household of type m	dwelling of type k'	dwelling of type k'	dwelling of type k'
living in zone i		living in zone i	living in zone i	in zone i' offered to	in zone i' offered to	in zone i' offered to
in dwelling of type k			in dwelling of type k	household of type m	household of type m	household of type m
	working in zone j	working in zone j	working in zone j	working in zone j	working in zone j	working in zone j
	looks in zone i'	looks in zone i'	looks in zone i'		living in zone i	living in zone i
	for dwelling of type k'	for dwelling of type k'	for dwelling of type k'			in dwelling of type k
	decision	decision	decision	decision	decision	decision
releases old dwelling			releases old dwelling			releases old dwelling
transaction completed						

Fig. 5. Market transactions in the housing market model.

Once the transaction has been completely defined, the migration decision is made. This is no question for outmigrant households, they do migrate. All other households compare their present housing situation with the situation they would gain if they accepted the transaction. It is assumed that they accept if they can significantly improve their housing situation. The definition of what is considered a significant improvement has to be determined by calibration. The measure of improvement is the difference between the satisfaction received by the present dwelling and the satisfaction expected from the dwelling offered.

If there is a significant improvement, the household accepts. In this case all necessary changes in R, H, H', H", and D are performed. Dwellings vacated with a move or an outmigration immediately reappear in the D matrix and are released again to the market.

If there is no improvement, the household declines. If the transaction type was "dwelling wanted", the household makes another try to find a dwelling, and with each attempt it accepts a lesser improvement. After a number of unsuccessful attempts the household abandons the idea of a move. If the transaction type was "dwelling for rent/sale", the landlord tries to find another household, but he does not reduce the rent during the market period. If a dwelling type in a zone has been declined by all household types, it is taken out of the market for this market period.

After successful or unsuccessful completion of a transaction the next transaction is selected. The market process comes to an end when there are no more households considering a move. It is assumed that this is the case when a certain number of transactions have been rejected. This number has to be determined by calibration to match the number of migrations produced by the model with the number of migrations observed in the region.

4. MODEL DATA AND CALIBRATION

The main data sources for the housing market model are tapes of the 1968 housing census and the 1970 census especially prepared for this project by the City of Dortmund. They are the basis for establishing the disaggregate (12o-type) distributions of households and housing and of the occupancy matrix of each zone. The model results are checked against spatially disaggregate population and housing data of the year 1977 also made available by the City of Dortmund.

The base year household distributions of the ten districts of Dortmund were retrieved from the census tapes containing individual data. However, for the 19 neighbouring communities, for which such tapes were not available, estimates based on one-dimensional distributions taken from statistical tables had to be made. A special estimation technique was developed to substitute the income information not contained in the census data. By this technique each household is associated with one of the four income groups depending on the employment status and completed education of its head, both which informations were available on the tapes (cf. Gnad, Vannahme, 1980).

Base year data of the housing stock were taken from the 1968 housing census. As with the household data, tapes containing information on a dwelling-by-dwelling basis were available for Dortmund, while some estimation of distributions had to be made for the neighbouring communities. All information needed to establish the 120-type housing distribution for each zone was contained on the tapes. However, the quality attribute had to be estimated as an aggregate of a number of dwelling attributes.

Establishing the base year occupancy matrix presented a special problem. The 1968 housing census contained detailed housing information, but only very limited information about households. The 1970 census contained detailed household, but no housing information. The problem was to match both kinds of census, although they were 18 months apart in time. The problem was solved by first generating for each zone a household-housing matrix from the 1968 data and then "blowing it up" to match the 1970 household distribution.

The major problems of model calibration, besides estimation of numerous demographic, technical, and monetary parameters, are connected with the *basic event probabilities* for the aging submodel and the *index of housing satisfaction* used in the migration submodel.

The basic event probabilities are partly linked to empirically well established demographic parameters and can be checked against exogenous population projections. Much more difficult is the estimation of probabilities for events like "new household of child", "rise of income", "decrease of income", "retirement", or "new job", for which only few data on the basis of household types exist. However, the only alternative to their approximation to one's best judgment would be to ignore them, which is no real alternative in a model based so much on household decisions.

Even more crucial is the estimation of the preference functions used to calculate the index of housing satisfaction for the migration submodel. There can be no doubt that the calibration of hundreds of utility functions and weights even for a past period of time, let alone their extrapolation into the future, heavily overtaxes the available data. But again, not to include them in the model would mean to ignore the essential variety of housing needs and tastes, which certainly would be the worse alternative.

Besides, this kind of model parameters has one great advantage: Their meaning can be communicated to everyone in everyday language, which makes them amenable to discussion and judgment. Consequently, formal estimation techniques in a strict statistical sense play only a minor role in the calibration of the preference functions, instead, many of the functions are determined by judgment, inferences, analogies, and careful checking of plausibility. The empirical foundations of this informal way of model calibration include the numerous surveys of regional and urban housing markets conducted in recent years which contain a wealth of material on migration motives and housing preferences.

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